Manipulation of the electronic density in the longitudinal phase space in synchrotron light sources

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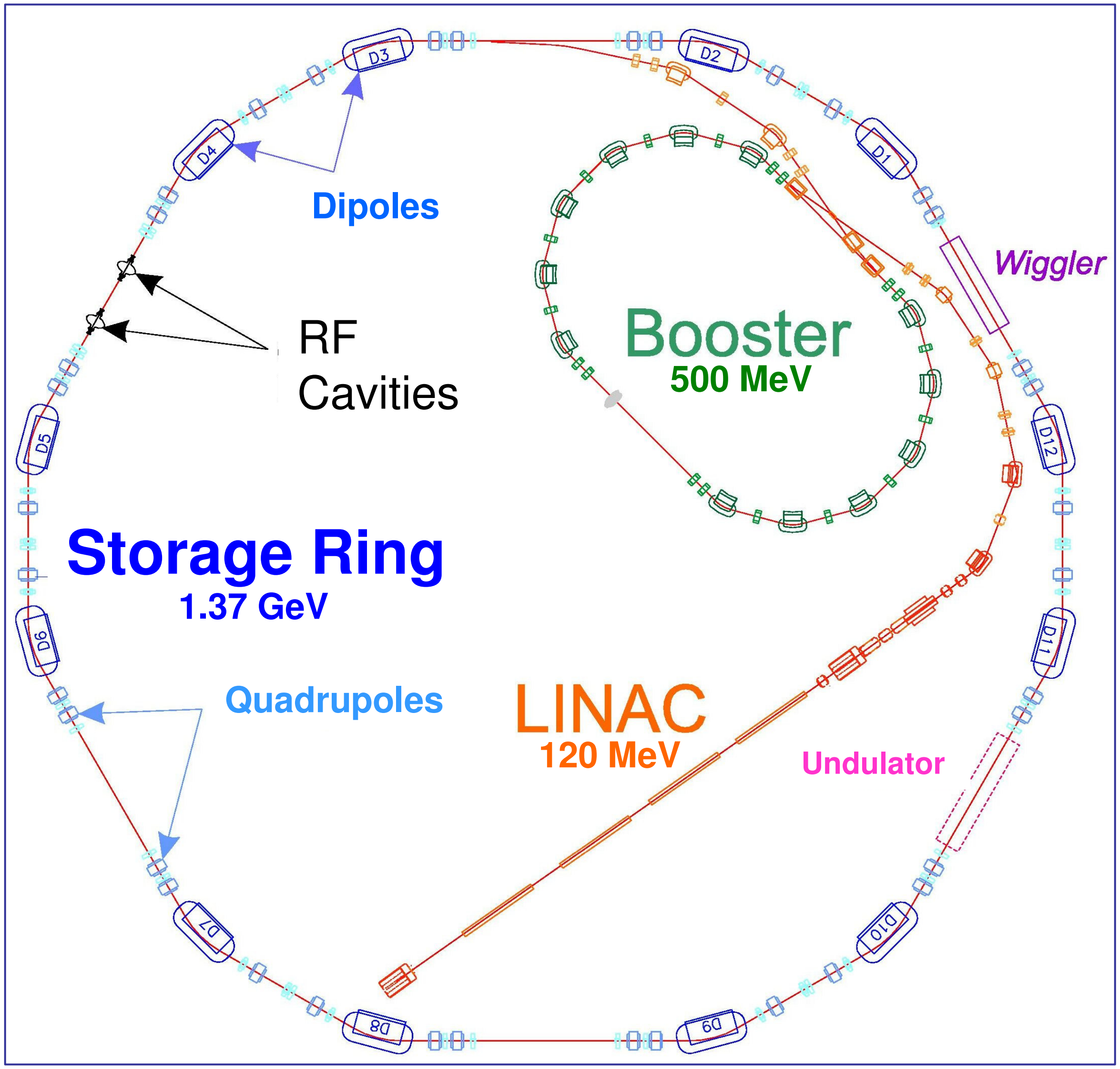
Outline

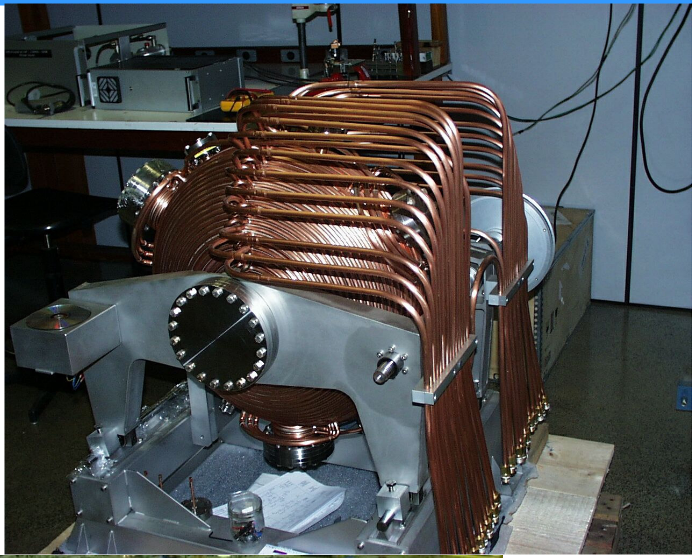
- Introduction:
  - The Brazilian synchrotron light source;
  - Figures of merit in light sources;
- Longitudinal instabilities at the Brazilian light source;
- Phase Modulation:
  - Theory
    - Equations of motion;
    - Effects of radiation damping;
    - Dynamics around fixed points and
    - Longitudinal BTF.
  - Simulation
    - Single and multibunch
  - Experiments
- Conclusions

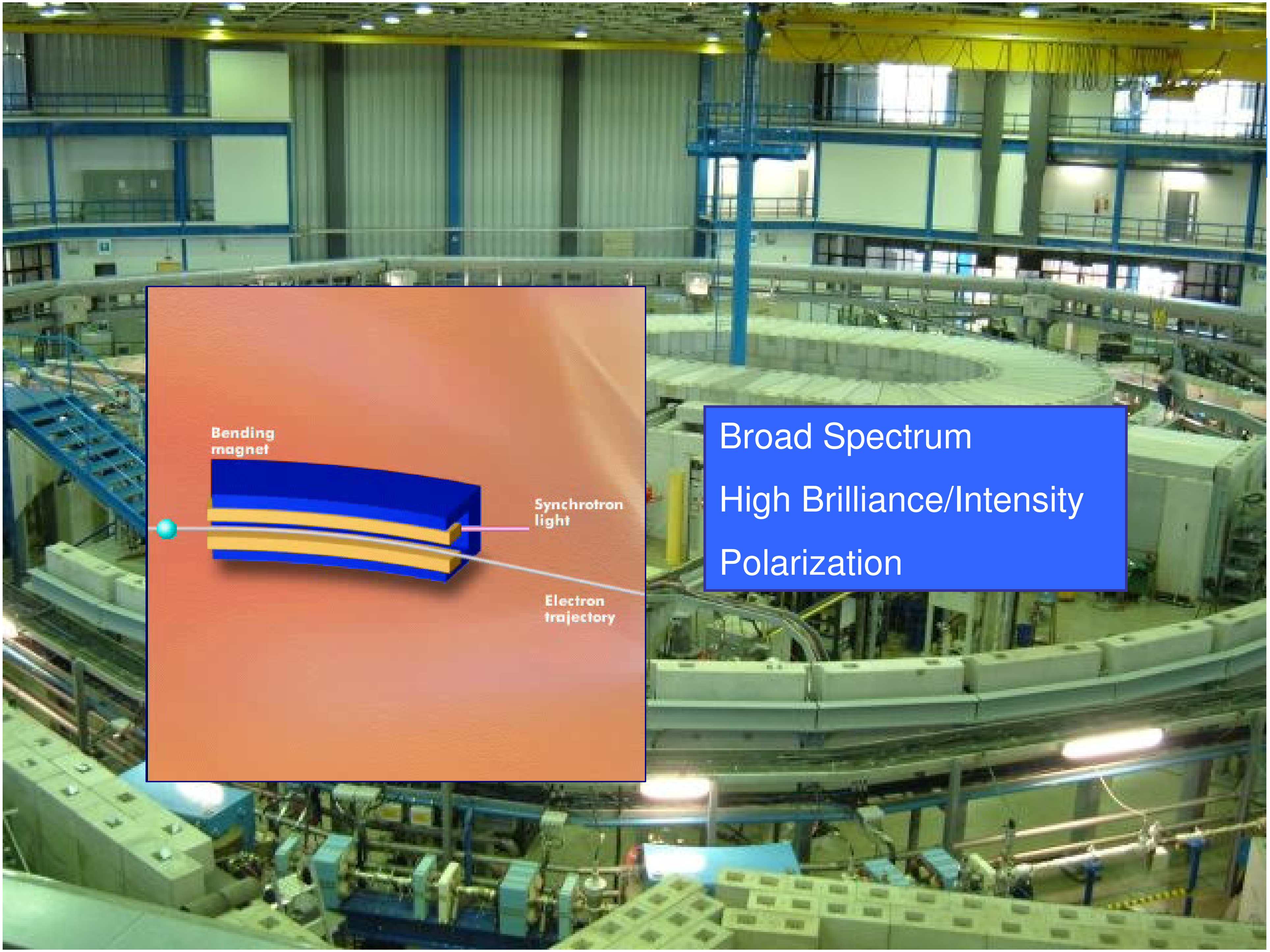
Introduction: The Brazilian synchrotron light source



Introduction: The Brazilian synchrotron light source

Energy, E	1,37	GeV
Initial Current, I	250	mA
Revolution frequency, f_0	3,22	MHz
# RF Cavities	2	
Power	100	kW
RF Frequency, f_{RF}	476,066	MHz
Harmonic Number, h	148	
Horizontal Tune, Q_x	5.27	
Vertical Tune, Q_y	2.17	
Emitance	100	nm-mrad
Coupling	0.3	%
Energy loss/turn, U_0	114	keV
Power radiated at the dipoles/100 mA, P_0	11,4	kW
Accelerating Voltage	500	kV
Synchrotron Frequency	26	kHz
Momentum compaction factor, α	$8,3\times10^{-3}$	





Broad Spectrum

High Brilliance/Intensity

Polarization

Introduction: Figures of merit in light sources

- Lifetime and Stability – figures of merit of 3rd generation light sources;
- Lifetime measures the loss rate of particles from the beam in time. A long lifetime is often desired since it leads to a reduction in thermal transients of the machine and beamlines which occurs at every injection.
- Collectives effects – can degrade the quality of the light delivered to the users.

Longitudinal instabilities at the Brazilian light source

- End of 2003 – Installation of a new RF cavity;
- Jan. 2004 – horrible synchrotron light quality delivered to the users;
- May 2004 – the source of the instability is a longitudinal HOM of the new RF cavity.

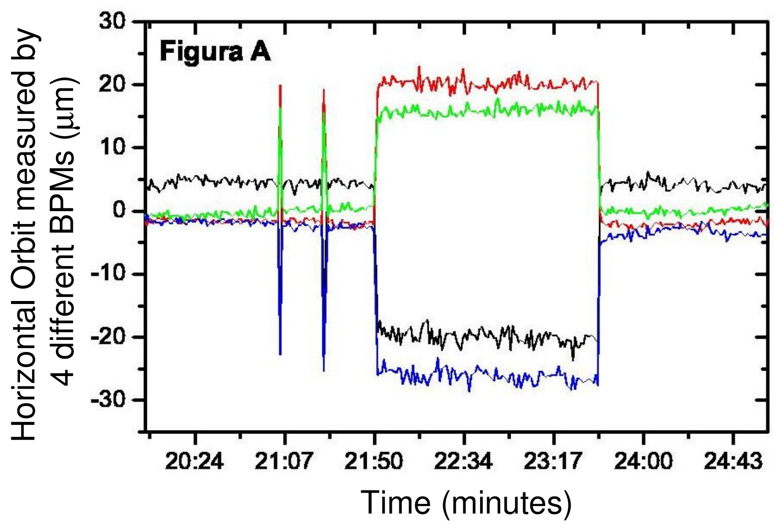
$$x_\varepsilon(s) = \eta(s)\delta + \eta_1(s)\delta^2$$

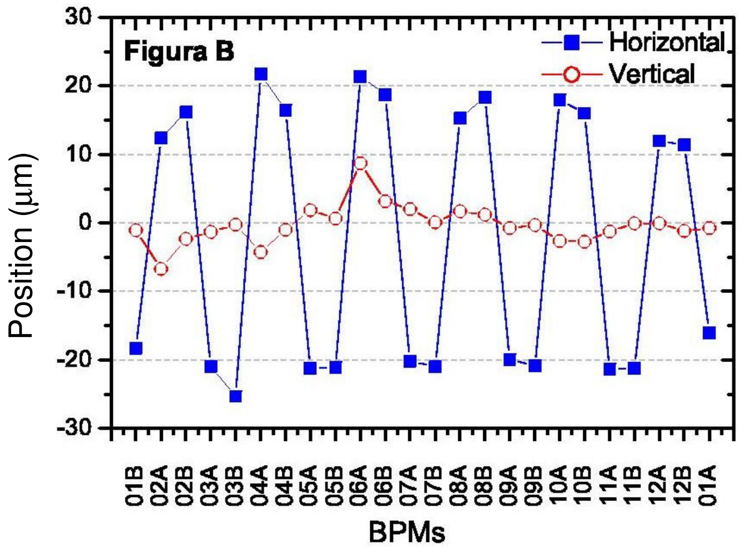
$$\delta_{HOM}(t) = \delta_0 \cos(\omega t)$$

$$x_\varepsilon \rightarrow \text{horizontal position}$$

$$\eta(s) \rightarrow \text{dispertion function}$$

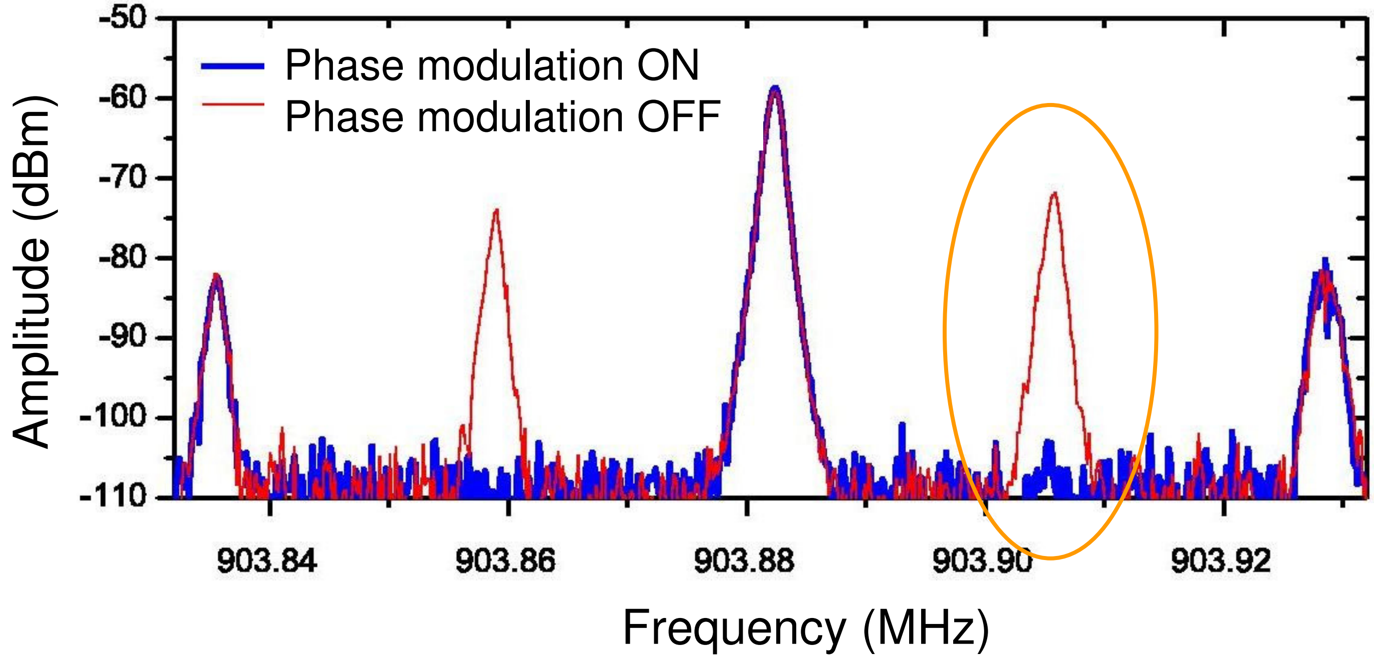
$$\delta \rightarrow \text{energy deviation}$$





Longitudinal instabilities at the Brazilian light source

- June 2004 – The instability is successfully suppressed using phase modulation of the RF fields near the second harmonic of the synchrotron frequency.



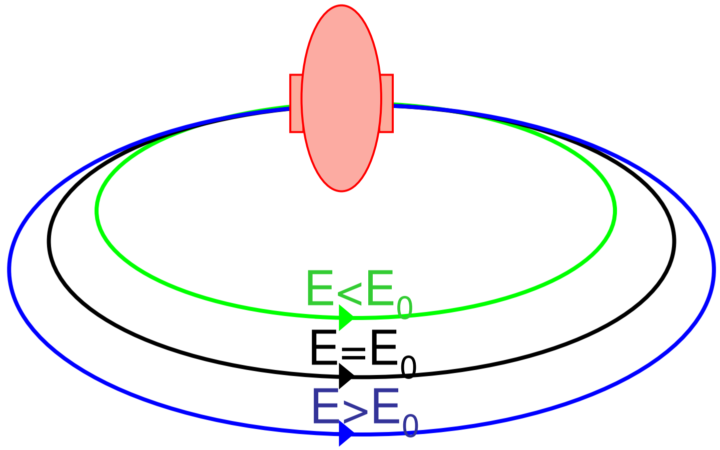
Overall Panorama

- Studies related to the effects of phase modulation:
  - Hadron machines [IUCF, APS, Fermilab, BNL];(H. Huang et al. Phys. Rev. E **48**, 4678 (1996), D. Li et al. Phys. Rev.E **48**, R1638 (1993) e D. Li et al., Nucl. Instrum. Methods Phys. Res. Sect. A **364**, p.205-223 (1995))
  - Electron machines [KEK, ALS, ELETTRA]:(S. Sakanaka, M. Izawa, T. Mitsuhashi, and T. Takahashi, Phys. Rev. ST Accel. Beams **3**, 050701 (2000) e J.M. Byrd, W. H. Cheng, and F. Zimmermann, Phys. Rev. E **57**, 4706 (1998))
    - Improve beam lifetime and
    - Suppression of CBM's.
- Lifetime – Effective increase in the bunch length leads to a reduction in particle density reducing the probability of electron-electron scattering (Toucheck effect).
- Stability – ?

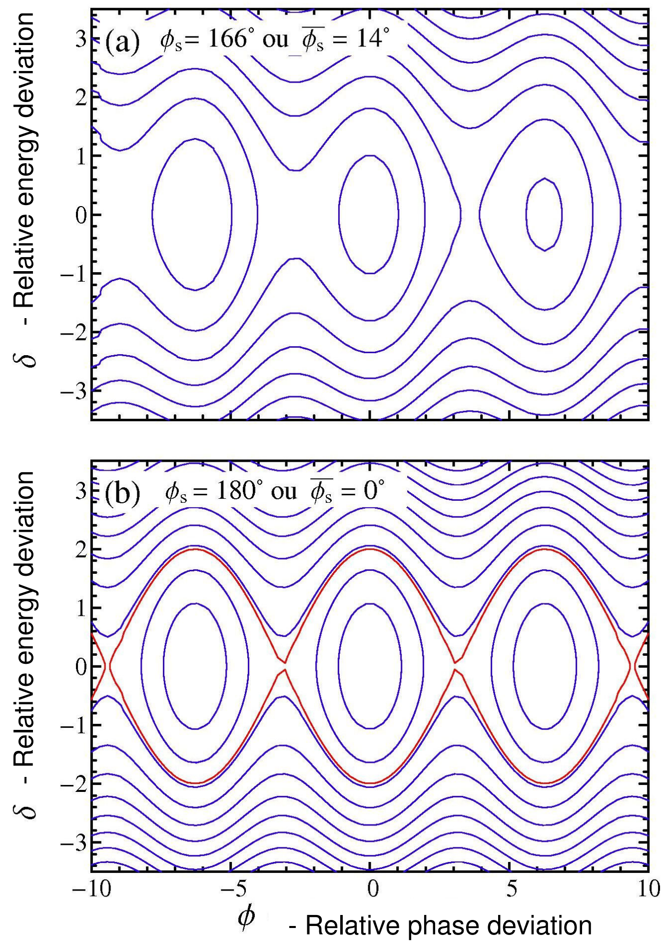
Longitudinal dynamics

- Equation of motion:

$$\frac{d^2\phi}{dt^2} + 2\gamma_d \frac{d\phi}{dt} + \frac{\omega_s^2}{\cos\bar{\phi}_s}\sin[\phi + \bar{\phi}_s] = 0$$



$$\frac{\Delta\tau}{\tau_0} = \eta \frac{\Delta p}{p_0}$$



Longitudinal dynamics

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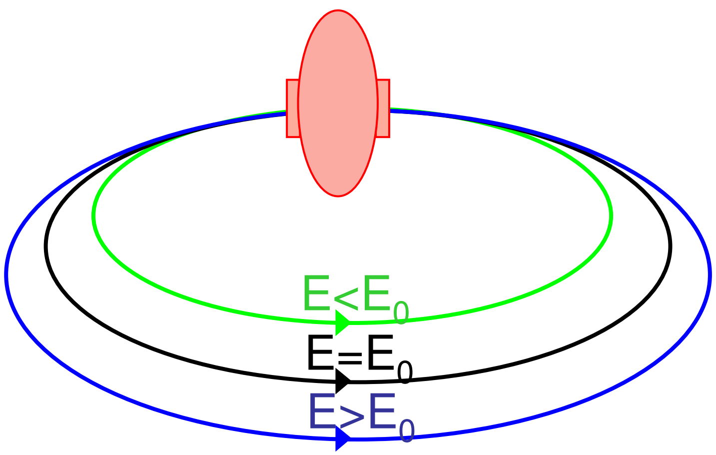
$$\frac{d^2\phi}{dt^2} + 2\gamma_d \frac{d\phi}{dt} + \frac{\omega_s^2}{\cos\bar{\phi}_s}\sin[\phi + \bar{\phi}_s + A_m \cos(\omega_m t)] = 0$$



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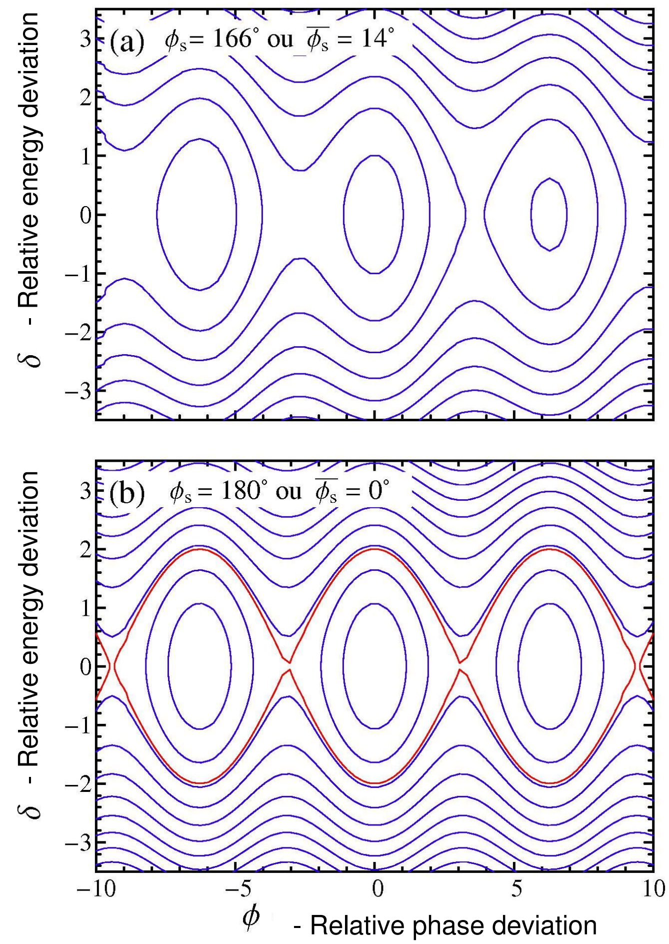


Longitudinal dynamics with phase modulation

Equation of motion with RF phase modulation:

$$\frac{d\phi}{dt} = -\eta\omega_{rf}\delta$$

$$\frac{d\delta}{dt} = \frac{eV_{rf}\cos(\phi + \phi_s + A_m\cos(\omega_m t)) - U_0}{T_0 E_0}$$

The Hamiltonian of the system is:

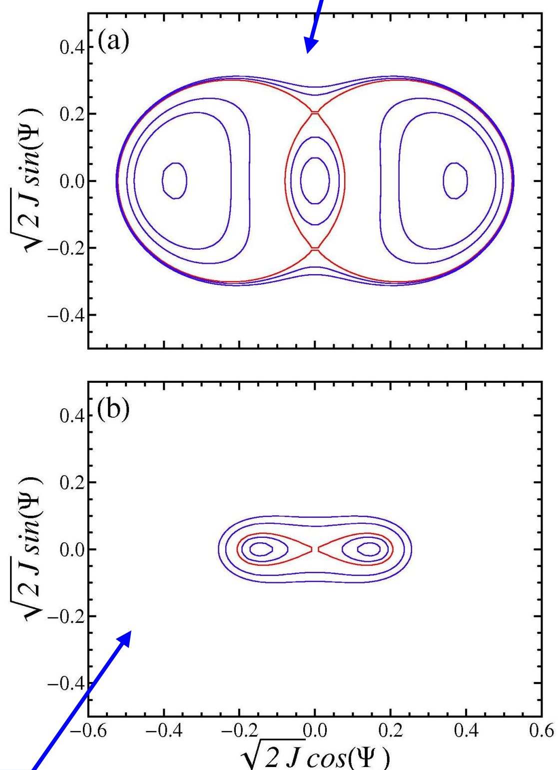
$$H(\delta,\phi) = \frac{\omega_s\delta^2}{2} + \omega_s\tan\bar{\phi}_s\sin[\phi + A_m\sin\omega_m t]$$
$$- \omega_s\cos[\phi + A_m\sin\omega_m t] - \omega_s\phi\tan\bar{\phi}_s$$

Changing to a reference frame which rotates with an angular velocity of $\omega_m/2$ and averaging the fast oscillating terms:

$$\langle H\rangle_t = \left(\omega_s - \frac{\omega_m}{2}\right)J - \frac{\omega_s J^2}{16} + \frac{\omega_s A_m\tan\bar{\phi}_s J}{4}\cos 2\Psi$$

term related to phase modulation

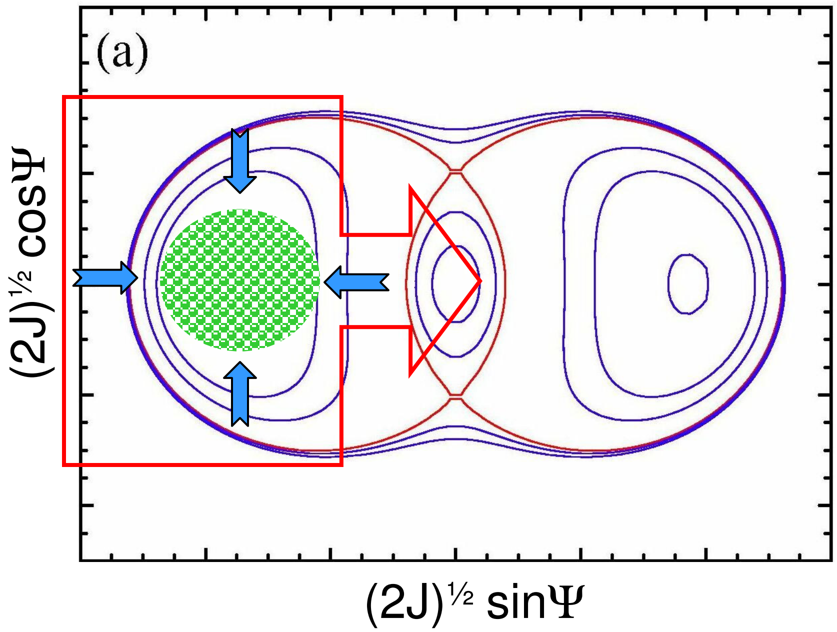
$\omega_m \le (2 - A_m|\tan\phi_s|/2)\ \omega_s$



$(2 - A_m|\tan\phi_s|/2)\ \omega_s < \omega_m \le (2 + A_m|\tan\phi_s|/2)\ \omega_s$

Effects of radiation damping

- It is necessary to take into account the effects of synchrotron radiation and how it can modify the results derived using the Hamiltonian formalism;
- We can divide the effect in tow parts:
  - 1 :Effect over the fixed points;
  - 2 : Effect over the electron distribution inside each island.



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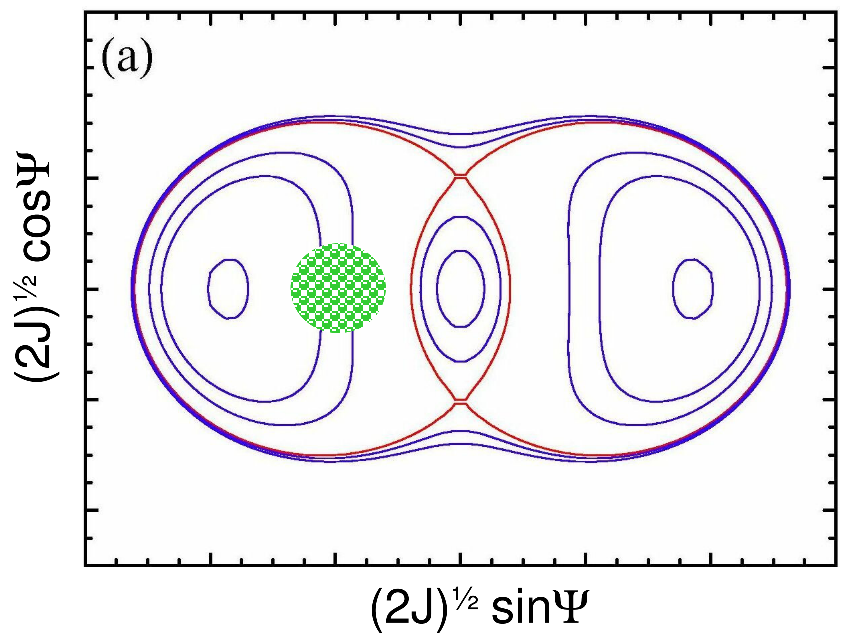


1: Effect over the fixed points

$$\frac{d^2\phi}{dt^2} + 2\gamma_d \frac{d\phi}{dt} + \frac{\omega_s^2}{\cos\bar{\phi}_s} \sin[\phi + \bar{\phi}_s + A_m \cos(\omega_m t)] = 0$$

$$A_m \geq \frac{4\gamma_d}{|\tan\phi_s|\omega_s} \approx 0.026 \text{ rad}$$

Amplitude limit: “competition” between the effects of damping due to radiation and excitation introduced by phase modulation.

$$A_m^{eff} = \sqrt{A_m^2 - \left(\frac{4\gamma_d}{|\tan\phi_s|\omega_s}\right)^2}$$

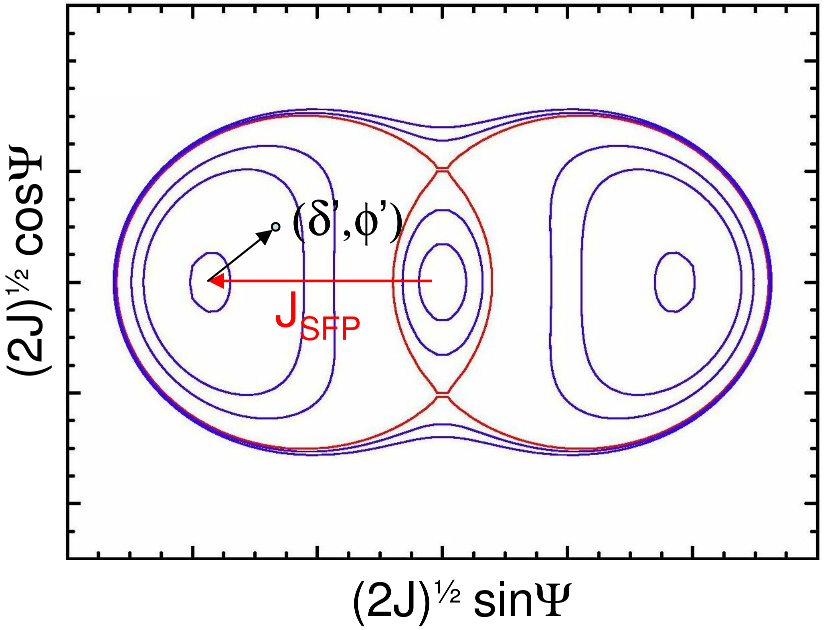
It is possible to include the effects of damping in the Hamiltonian treatment defining a new effective modulation amplitude.

2: Effect over the electron distribution inside each island.

Expanding the complete Hamiltonian around each fixed point:

$$H'(\delta',\phi') = \frac{A}{2}\delta'^2 + \frac{B}{2}\phi'^2 - \frac{\omega_s\sqrt{2J_o}}{16}\phi'^3 - \frac{\omega_s}{16}\left(\frac{\phi'^4+\delta'^4}{4}\right) - \frac{\omega_s\varepsilon}{24}\left(\frac{\phi'^4-\delta'^4}{4}\right)$$

$$\omega(\hat{\phi}') = \omega_o\left(1 - \frac{3\omega_s}{16}\,\frac{A^2+B^2}{A^2|B|}\,\frac{\hat{\phi}'^2}{8}\right)$$



where:

$$\varepsilon = A_m \tan\bar{\phi}_s$$

$$A = A(\varepsilon,\omega_m,\omega_s)$$

$$B = B(\varepsilon,\omega_m,\omega_s)$$

rms energy and phase spread:

$$\sigma_\delta = \sqrt{\frac{\kappa}{\gamma_d}} \qquad \sigma_\phi = \sqrt{\frac{A}{B}}\sigma_\delta = \sqrt{\frac{A\kappa}{B\gamma_d}}\ ,$$

$$\kappa = 4\times10^{-4}\,s^{-1} \qquad \gamma_d = 250\,s^{-1}$$

2: Effect over the electron distribution inside each island.

Expanding the complete Hamiltonian around each fixed point:

$$H'(\delta',\phi') = \frac{A}{2}\delta'^2 + \frac{B}{2}\phi'^2 - \frac{\omega_s\sqrt{2J_o}}{16}\phi'^3 - \frac{\omega_s}{16}\left(\frac{\phi'^4+\delta'^4}{4}\right) - \frac{\omega_s\varepsilon}{24}\left(\frac{\phi'^4-\delta'^4}{4}\right)$$

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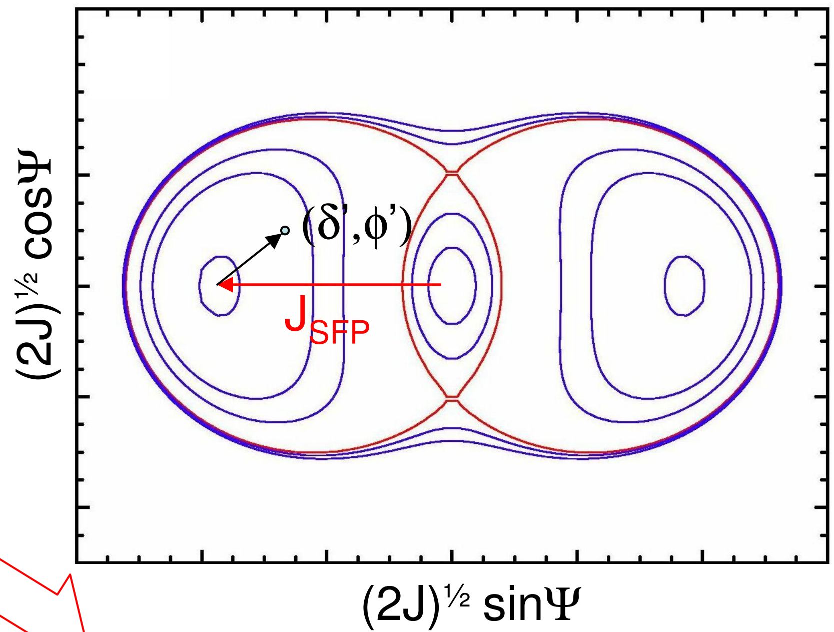
where:

$\varepsilon = \mathrm{A_m}\tan\bar{\phi}_s$

$A = A(\varepsilon,\omega_m,\omega_s)$

$B = B(\varepsilon,\omega_m,\omega_s)$

+ Fokker-Planck



rms energy and phase spread:

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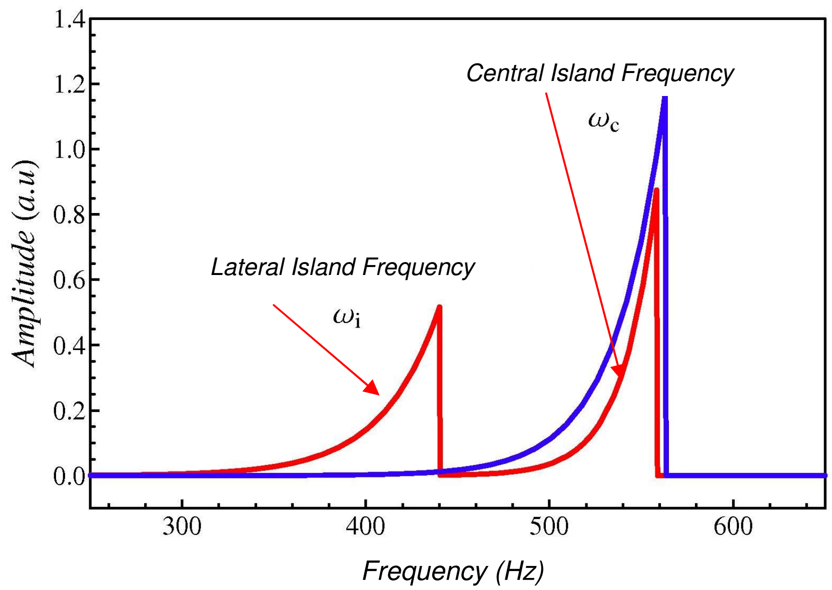
$\kappa = 4\times10^{-4}\,s^{-1} \qquad \gamma_d = 250\,s^{-1}$

$$\boxed{\frac{d\Psi}{dt} = 2\gamma_d\psi + \left(\gamma_d r + \frac{\kappa}{r}\right)\frac{\partial\Psi}{\partial r} + \kappa\frac{\partial^2\Psi}{\partial r^2}}$$

Landau Damping

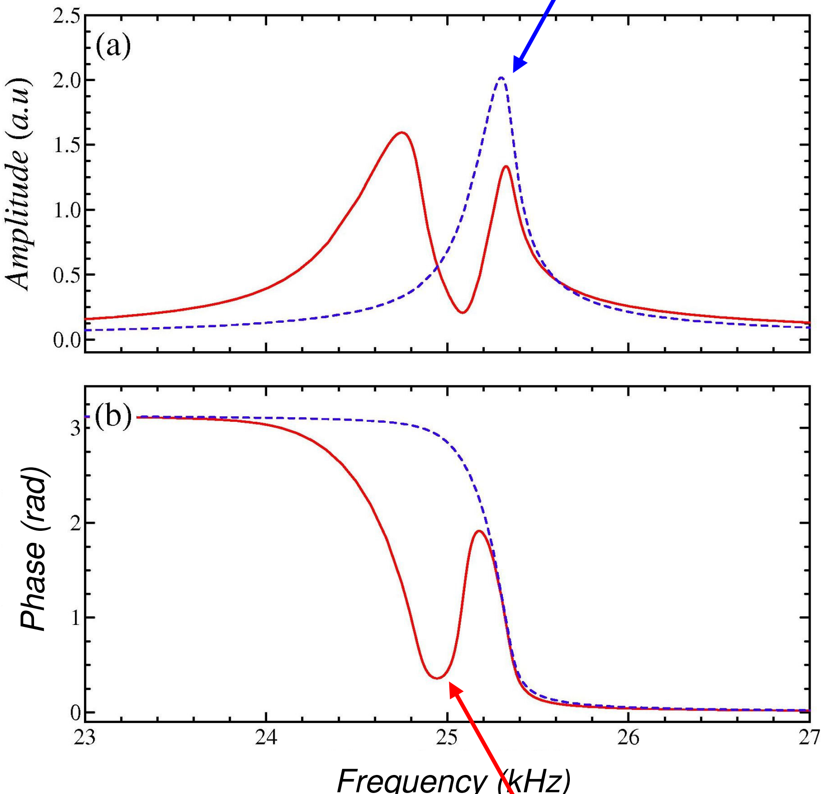
$$\tau \approx \frac{1}{\Delta\omega/2\pi + 1/\tau_{rad}}$$

- Frequency spread:
  - $\tau_{rad} \approx 4$ ms;
  - $\Delta\omega_{nat}/2\pi \approx 160$ Hz;
  - $\Delta\omega_{mod}/2\pi \approx 490$ Hz.
- Growth and damping times:
  - $\tau_{nat} \approx 3$ ms;
  - $\tau_{mod} \approx 2.2$ ms;
  - $\tau_{L1} \approx 3.3$ ms.



Longitudinal BTF

Without phase modulation



With phase modulation

Assuming that the eletronic distribution around each stable fixed point is a gaussian distribution:

$$\ddot{\tau} + 2\gamma_d \dot{\tau} + \omega_0^2 \tau = F_0 e^{-j\Omega t}$$

$$\underbrace{\Psi(r,\theta,t) = \Psi_0(r) + \Psi_1(r) e^{j(\Omega t - \theta)}}_{\Downarrow}$$

$$\bar{\tau}(t) = \int_0^{2\pi}\int_0^{\infty} r^2 \cos\theta \, dr d\theta \, \Psi(r,\theta,t)$$

$$\bar{\tau}(t) = \frac{F_0}{2\omega_c} e^{-j\Omega t} \underbrace{\left[N_c I_c(\Omega) + N_i \frac{\omega_c}{\omega_i} I_i(\Omega) \right]}_{\text{Longitudinal Transfer Function -I}(\Omega)}$$

$$I_n(\Omega) \equiv \pi \int_0^{\infty} \frac{r^2 dr}{\Omega - \hat{\omega}(r)} \frac{\partial \Psi_0(r)}{\partial r} \quad e \quad N_c + N_i = 1$$

$$\hat{\omega}(r) = \omega(r) - j\gamma_d$$

Stability Diagram

$$\frac{d^2\tau}{ds^2}+\frac{\omega_s^2}{v^2}\tau=-\frac{je^2N\eta\omega_{rev}\bar{\tau}}{E_0C^2}Z_{//}$$

Integrating over the whole electron distribution and using the ansatz

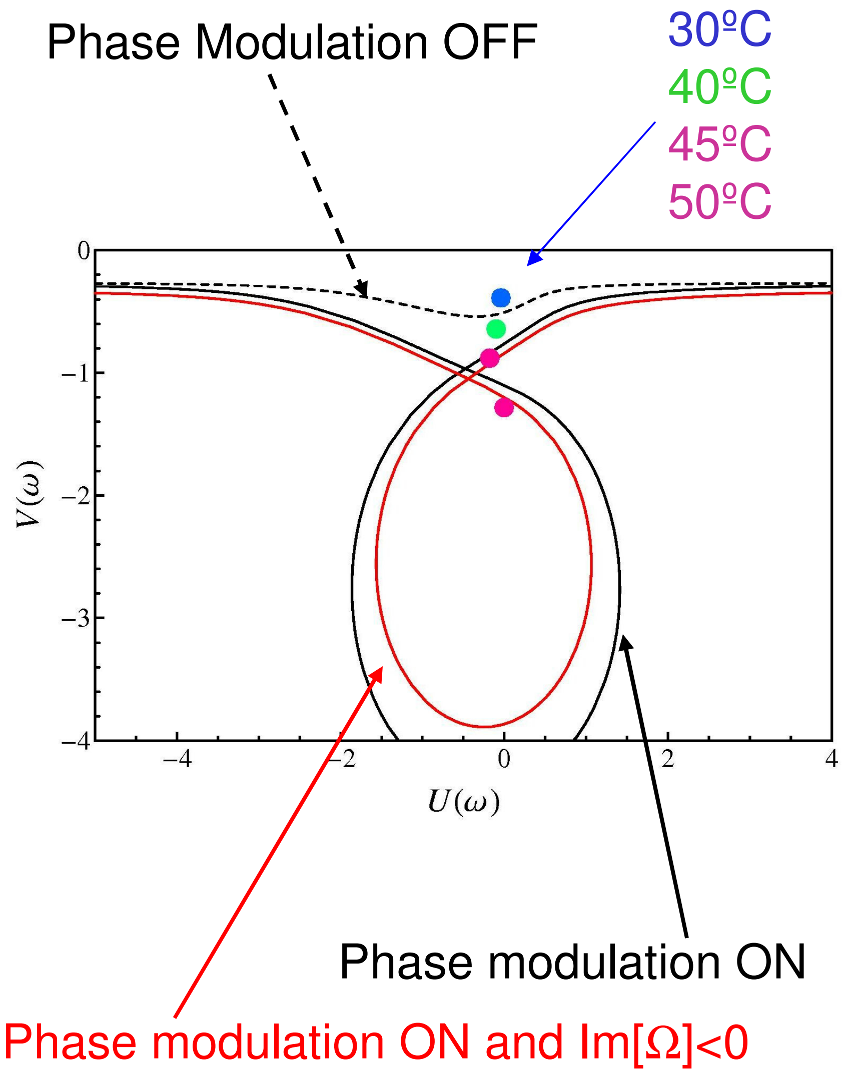
$\bar{\tau}(s)=Be^{j\omega s/v}$,

we can define the deviation of the coherent oscillation frequency:

$$\Delta\omega_{coh}\cong\Omega-\omega_0=-\frac{je^2N\eta\omega_{rev}}{2\omega_0EC^2}Z_{//}$$

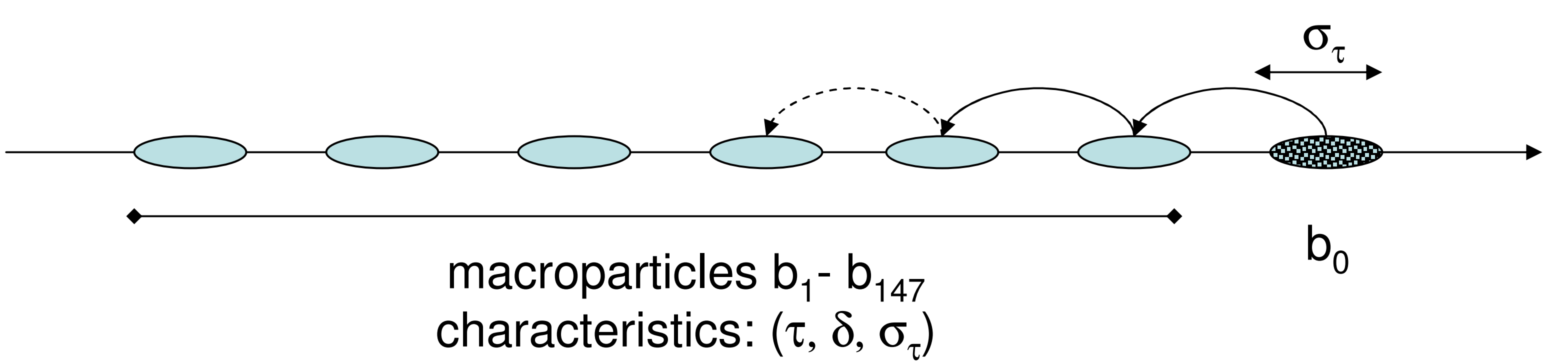
$\mathrm{Im}[\Omega]>0\Rightarrow Stability$

$$\boxed{U+jV=-j(\Delta\omega_{coh})=\frac{j}{I(\Omega)}}$$

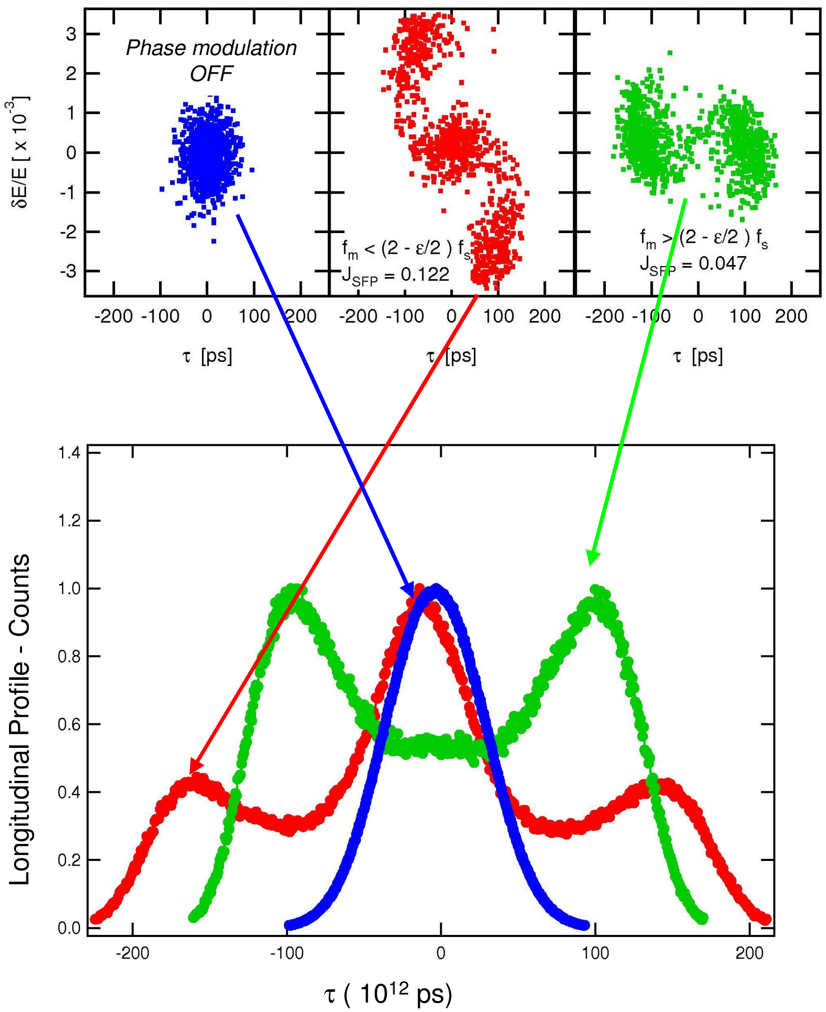


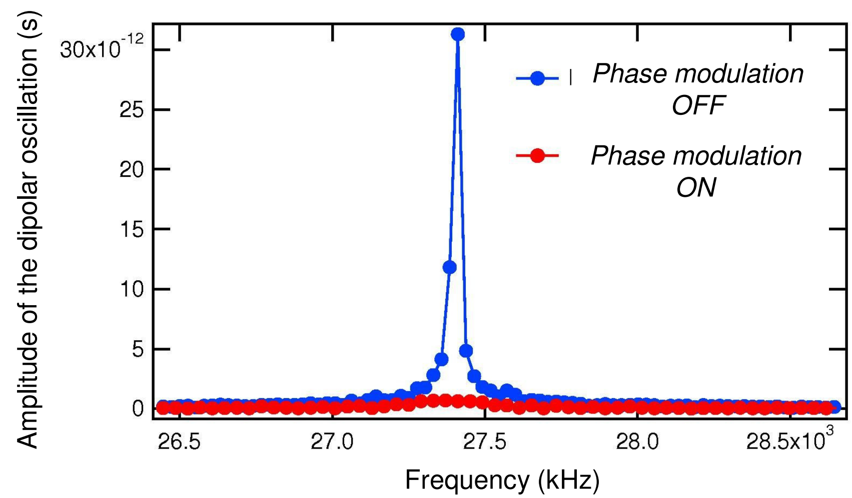
1D Longitudinal Simulation Code

- 1D code: longitudinal part only;
- Takes into account:
  - Damping and excitation due to synchrotron radiation and all nonlinearities of longitudinal dynamics;
  - Beam loading of the fundamental and L1 (longitudinal HOM) modes;
  - Phase modulation.
- Single bunch simulation: 1 bunch of 1000 macroparticles;
- Multibunch simulation: 147 bunches (macroparticles) and 1 bunch of 1000 macroparticles.



Single Bunch Simulation Results

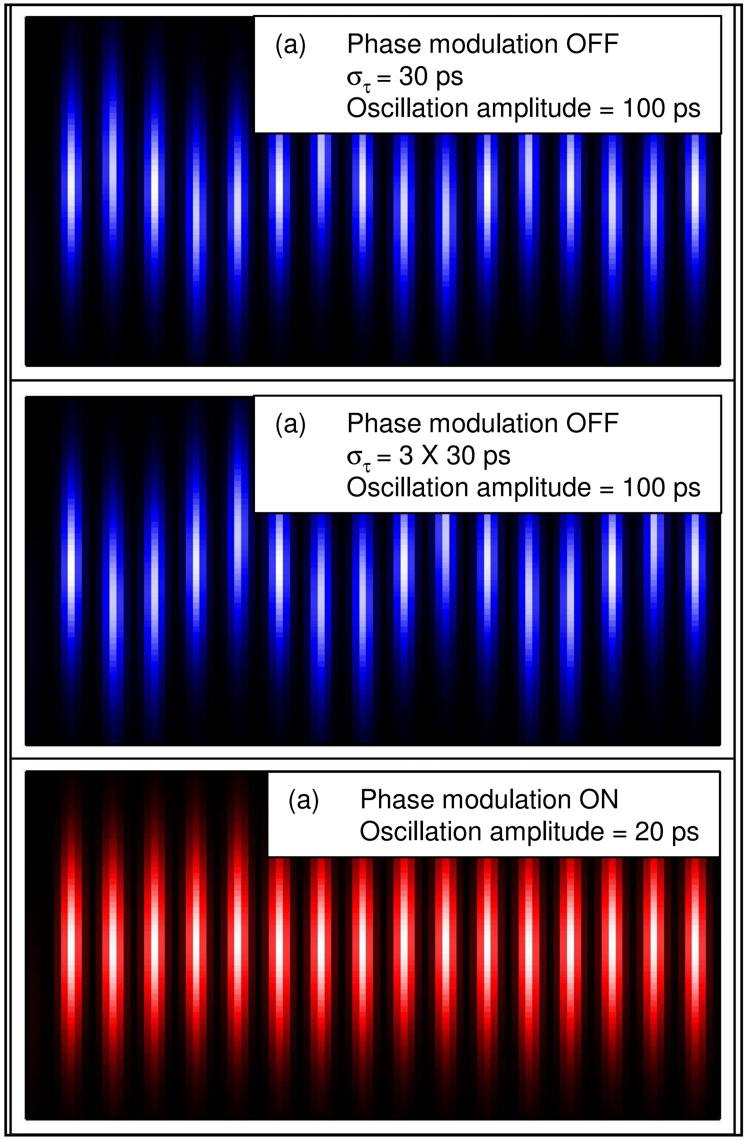




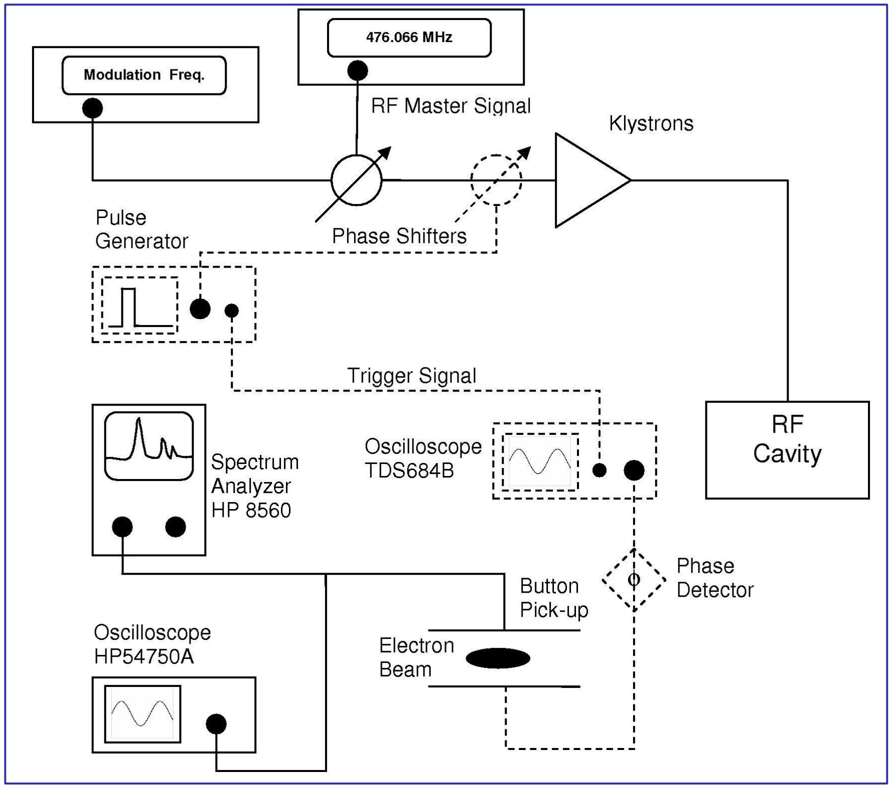
Excitation of a dipolar oscillation
in a single bunch with 30 mA.
Curently stored single bunch current: 10 mA.

Multibunch Simulation Results

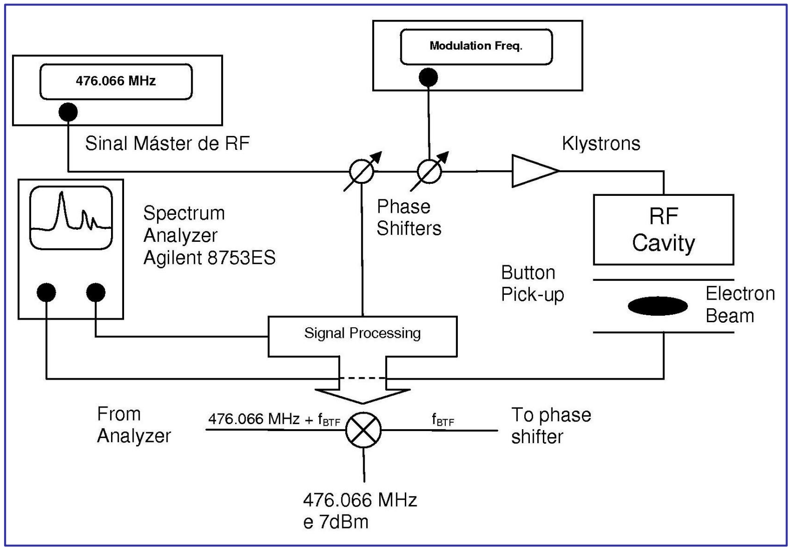
- Phase modulation OFF + HOM in the RF cavity;
- Phase modulation OFF + HOM in the RF cavity, also the bunch length is 3 times the natural length;
- Phase modulation ON + HOM : damping.



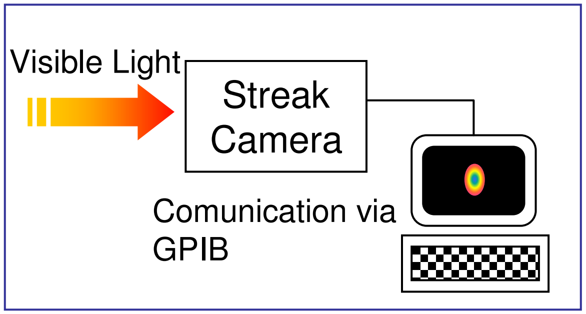
Experimental Setups



Experimental setup used to measure damping, spectrum and longitudinal profile.

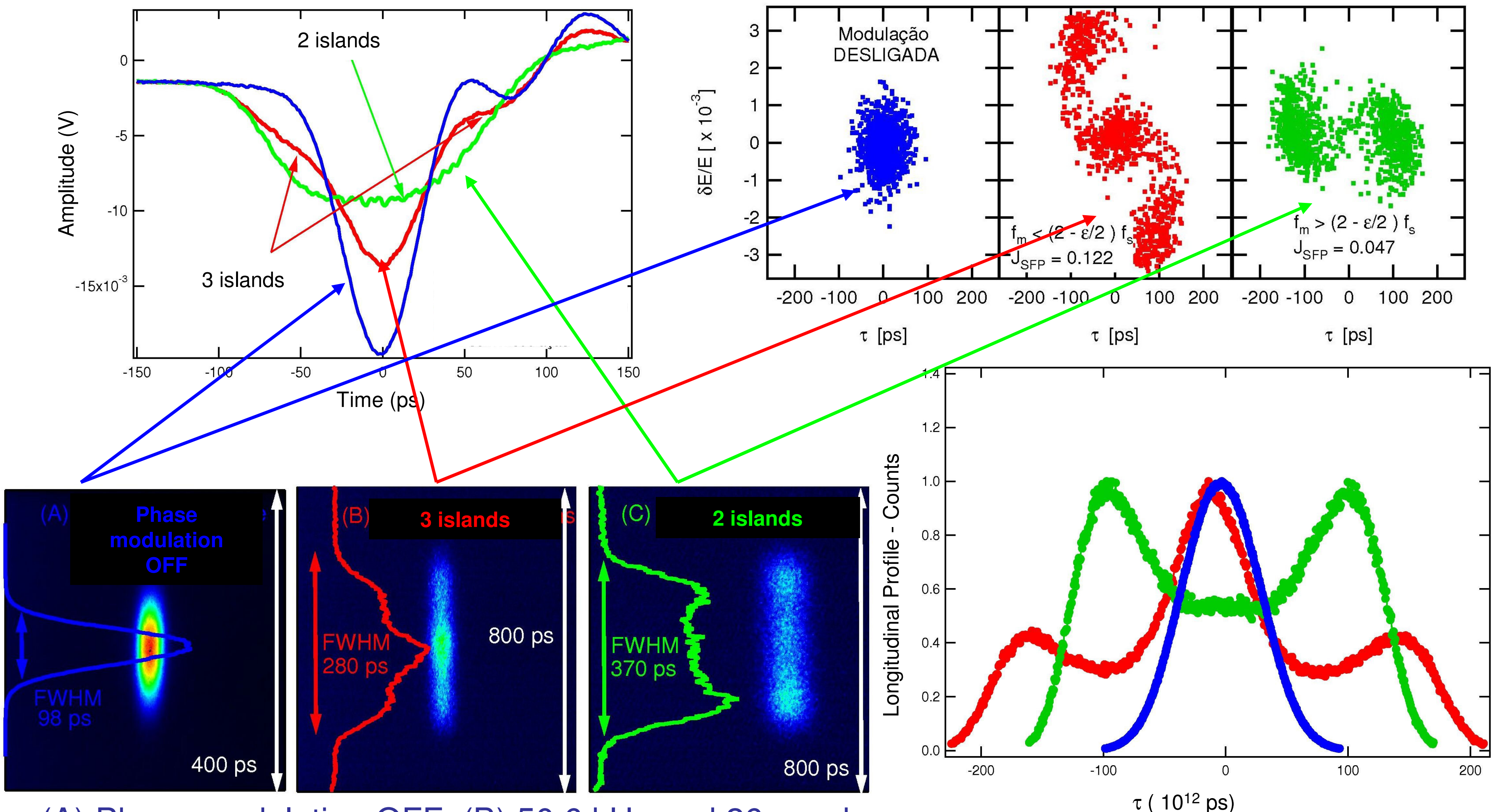


Experimental setup used to measure the longitudinal BTF.



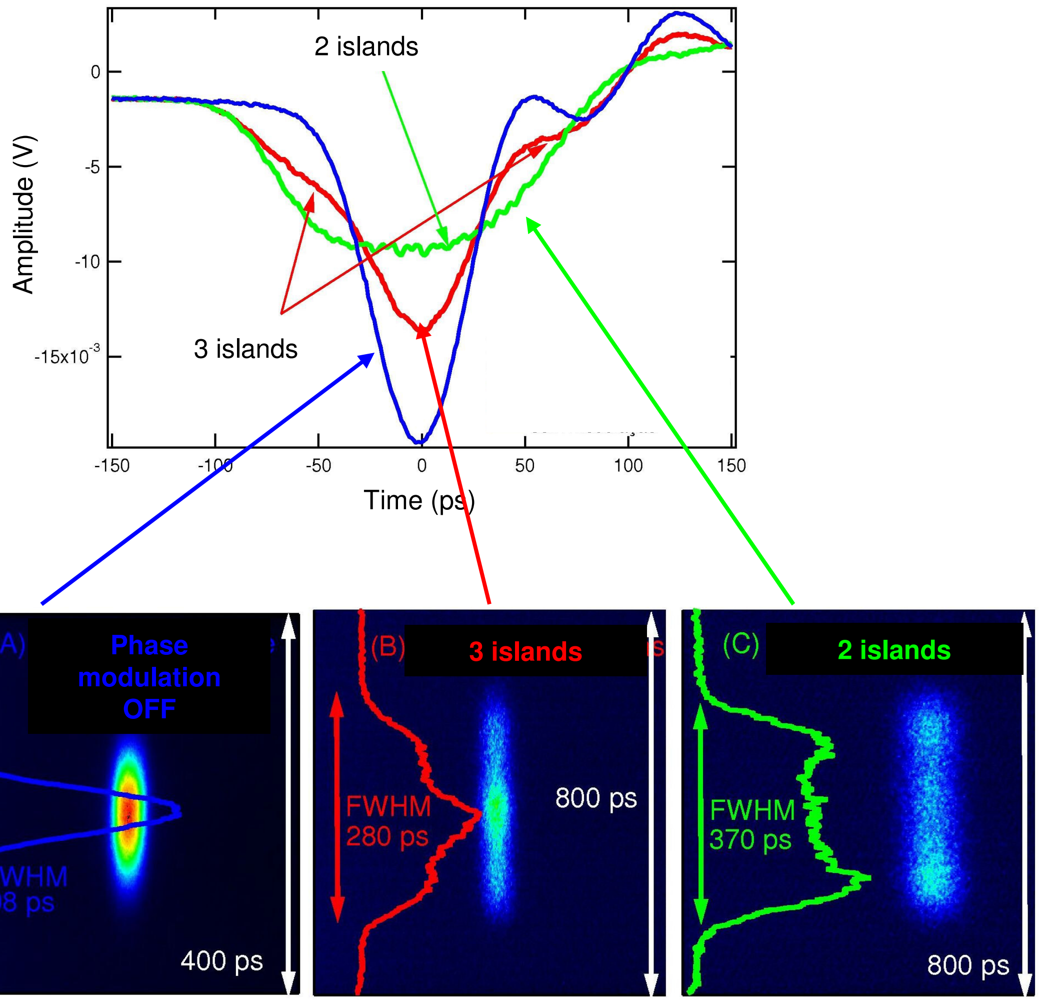
Experimental setup used to measure the longitudinal profile.

Island Formation



(A) Phase modulation OFF, (B) 50.6 kHz and 26 mrad
(C) 50.6 kHz and 46 mrad

Island Formation



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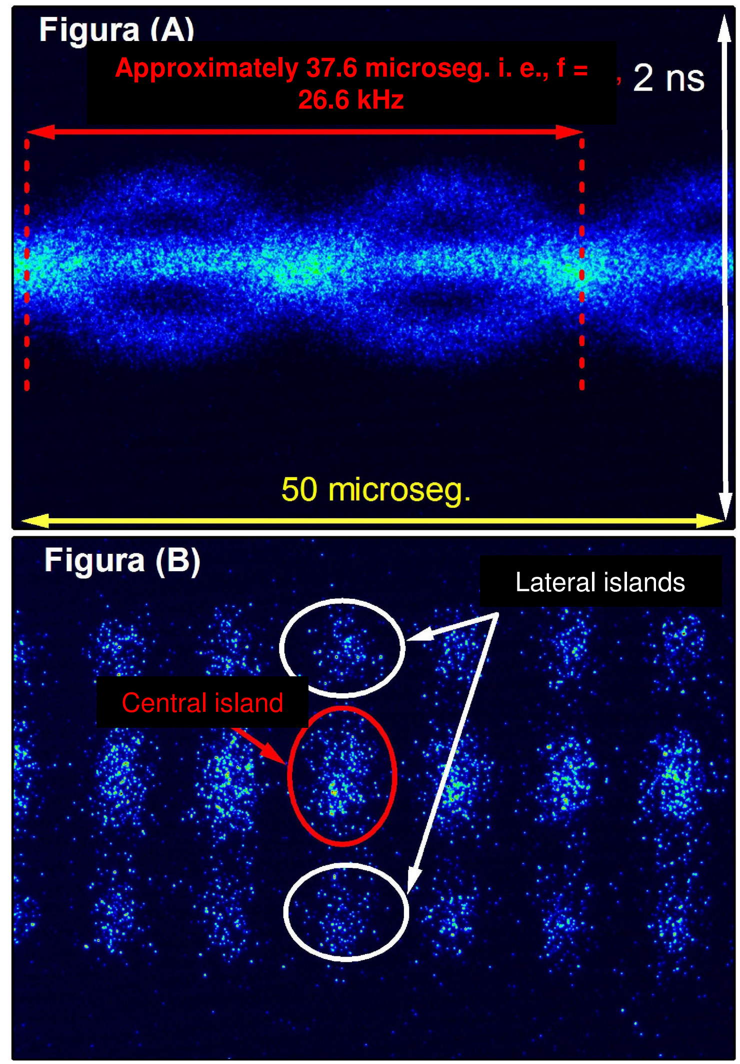
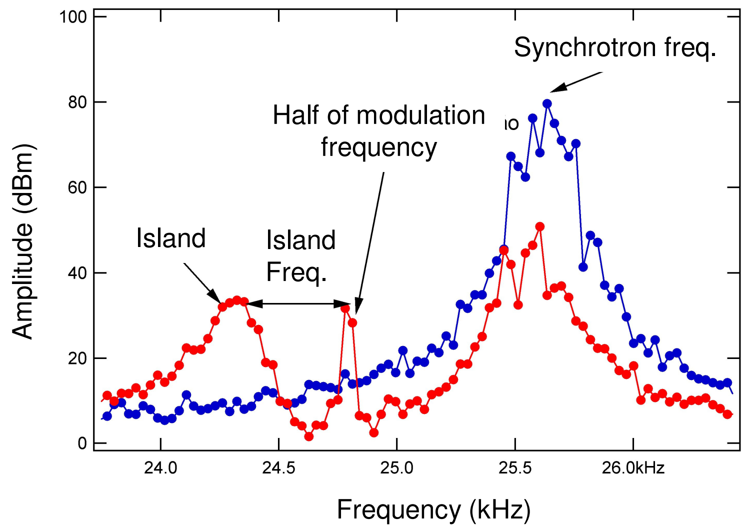
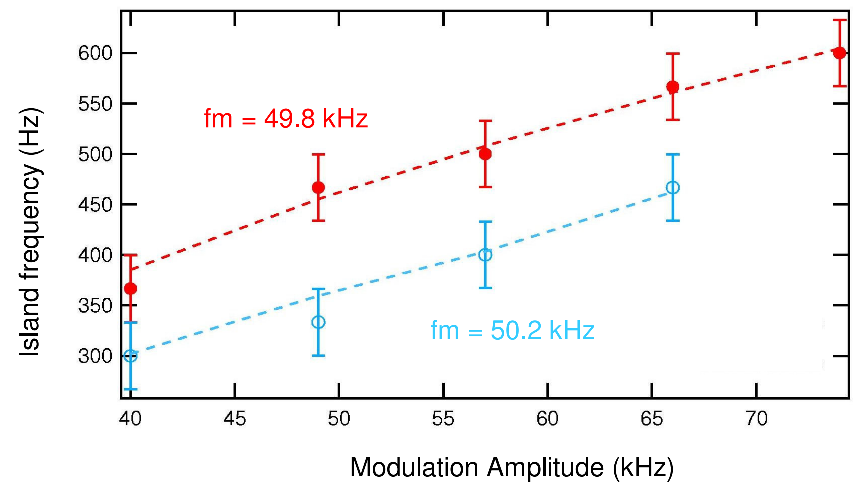


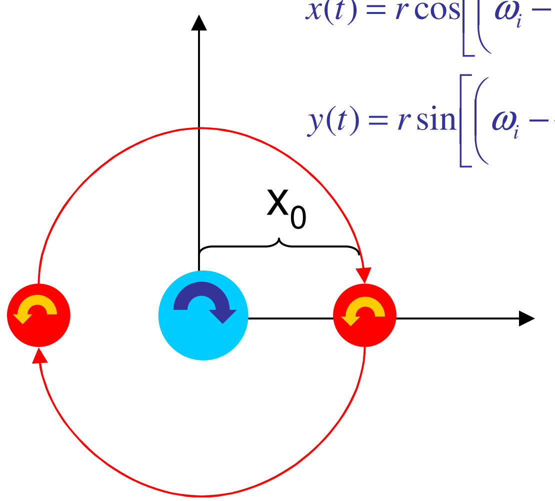
Frequency of oscillation around the fixed points

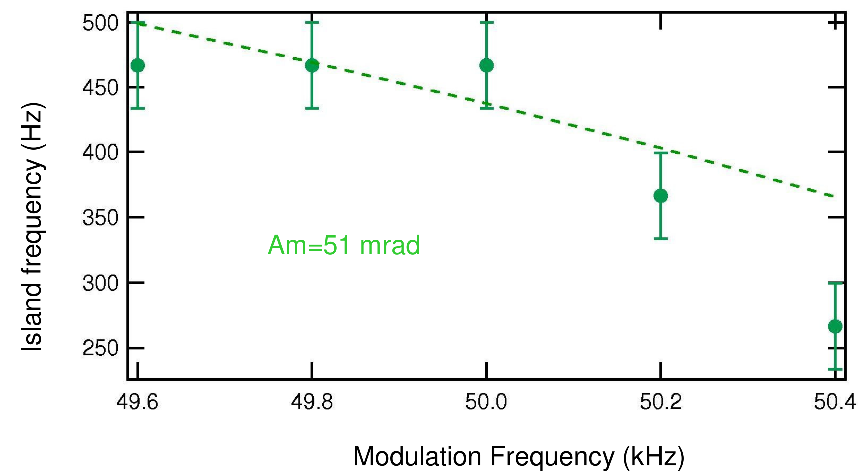




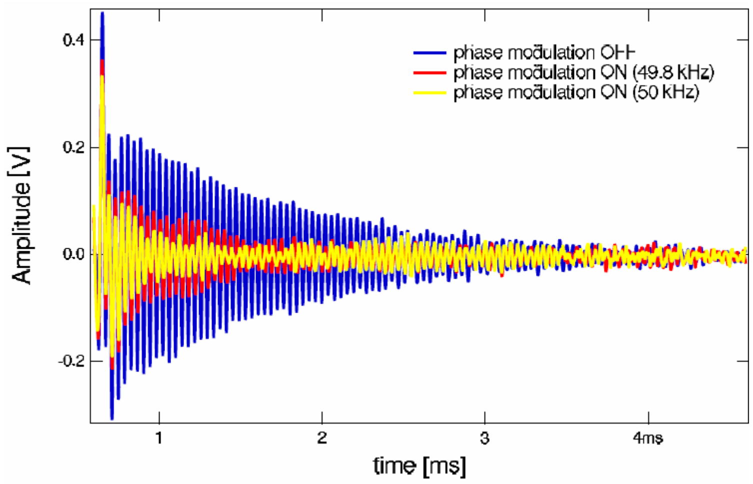
$$x(t) = r\cos\left[\left(\omega_i - \frac{\omega_m}{2}\right)t\right] + x_0\cos\left(\frac{\omega_m}{2}t\right)$$

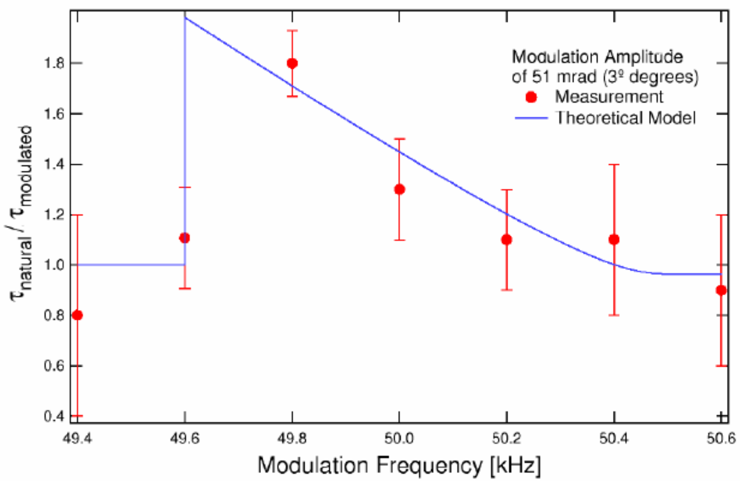
$$y(t) = r\sin\left[\left(\omega_i - \frac{\omega_m}{2}\right)t\right] - x_0\sin\left(\frac{\omega_m}{2}t\right)$$





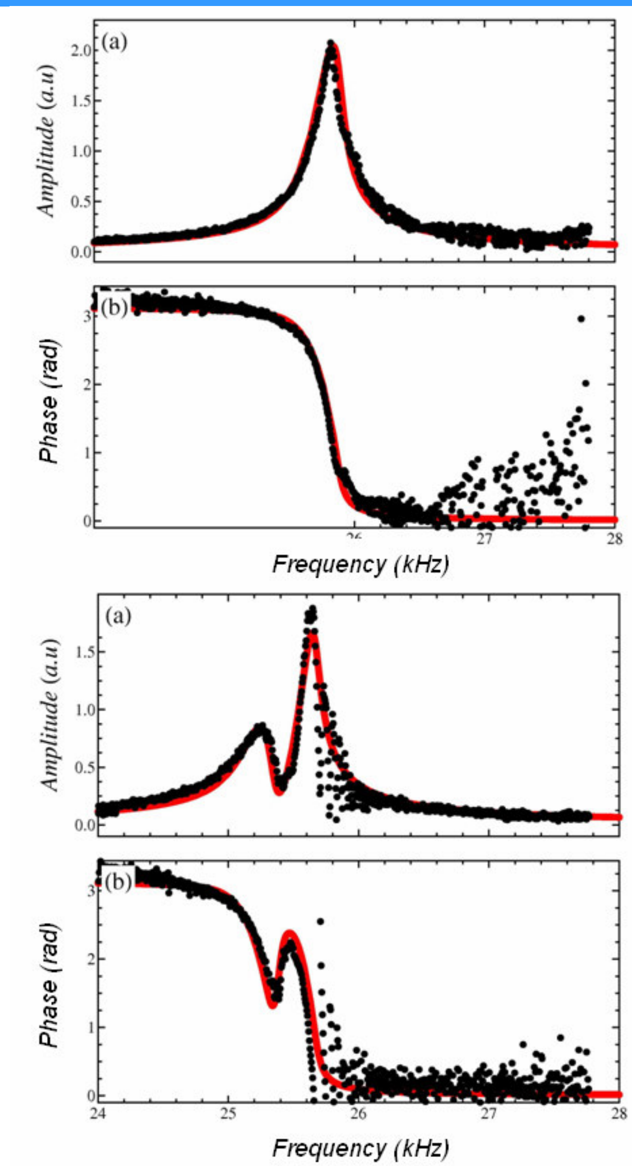
Damping Measurement



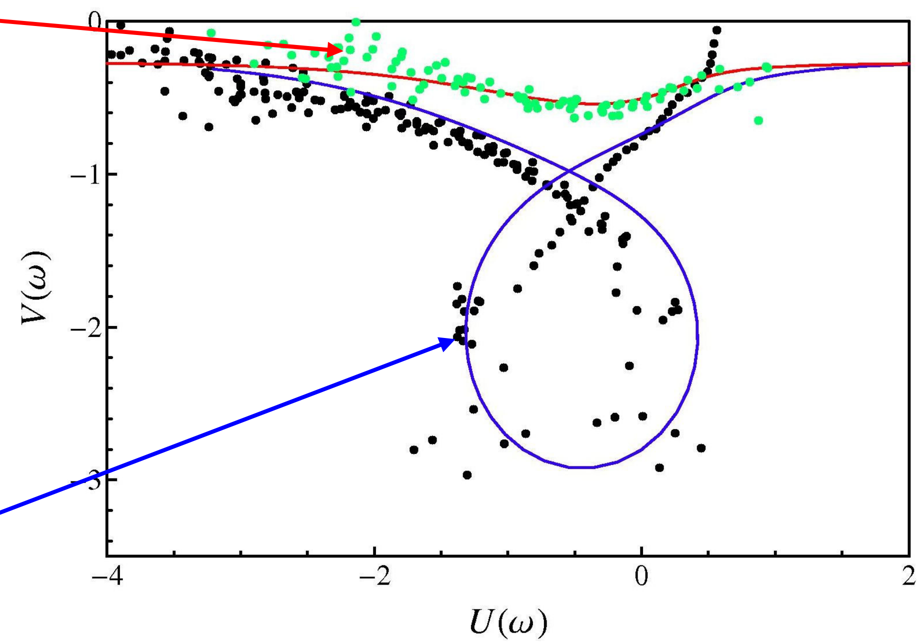


- Damping of synchrotron oscillations;
- Phase modulation is effective in a narrow region about 400 Hz wide;
- Maximum frequency:
  - $f_m = (2+A_m \tan\phi_s/2) f_s$;
  - theory = 50.3 kHz;
  - measurement ≈ 50.2 kHz;

Longitudinal BTF and stability diagram



Phase modulation OFF



Phase modulation ON

Conclusions

- Phase modulation can damp some unstable modes excited by HOMs of the RF cavities via Landau damping (increase in the synchrotron frequency spread);
- There is the formation of 2 or 3 stable islands in the longitudinal phase space as a function of the modulation frequency applied;
- The theoretical model can predict some of the main features of the modified longitudinal phase space including the frequency of the islands, the size and the form of the BTF response;
- It was also observed an increase of the beam lifetime due to the use of phase modulation by 30% in multibunch mode and 100% in single bunch mode;
- Phase modulation also increases the total energy spread of the bunches and it can be a problem when there are insertion devices such as undulators installed in the storage ring (specially when the emittances are small).
- Apart from other methods, phase modulation on the second harmonic of the synchrotron frequency is an effective, cheap, easy to implement and a reliable way to stabilize the beam.